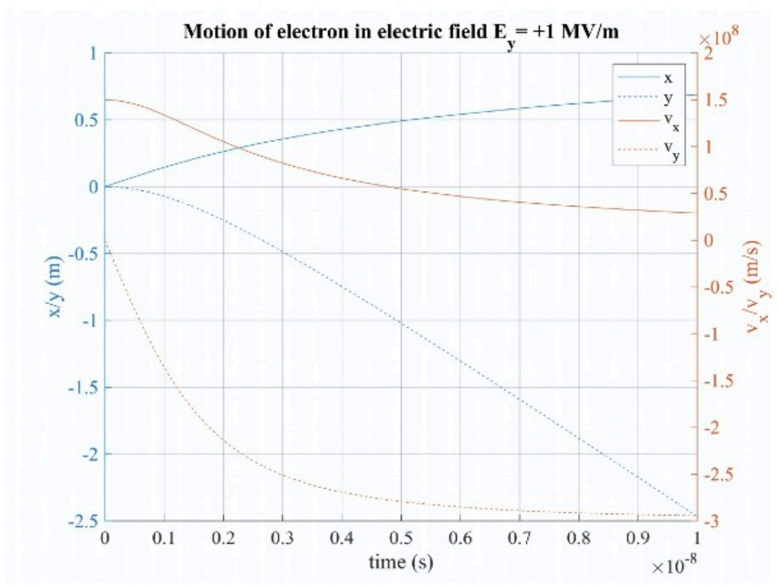


Notice that the x-component of velocity, subject to no force, nevertheless changes - the x-component of momentum is constant but the speed is changing due to acceleration in the y direction - and approaches zero as $v_y \to -c$.

Relativistic constant acceleration with MATLAB for 1st year physics students

How long in proper time would it you take to get to the nearest star subject to constant acceleration, without blacking out? What does a relativistic 2d ballistic motion trajectory look like? The motion of a particle accelerated to relativistic speed is examined in a MATLAB Live Script for first year physics students.

Duncan Carlsmith, WAPT, 7 Nov 2020

1st-year computational curriculum

- **What**: Physics 247-8, Introductory Physics using MATLAB, 5-cr, 2-sem, automatic honors, for ~80 <u>first year</u> Physics, Astronomy, and Applied Math Engineering and Physics (AMEP) students
- **How**: Tutorials with Live Scripts and integrated MATLAB analysis of laboratory data.
- **Topics**: Modeling, simulation, visualization, signal analysis, numerical techniques, symbolic algebra, intermediate and advanced mathematics, probability and statistics, and statistical analysis and interpretation of data.
- **Why**: A foundation for computation throughout the undergraduate physics curriculum.

Learning goals

- Understand and apply principles of physics…
- Innovate and apply.
- Own measurements and analysis.
- Understand and apply hardware and software technologies.
- Discover and use current research and open data.
- Think computationally and scientifically (model, test…).
- Collaborate and communicate!

1st semester syllabus

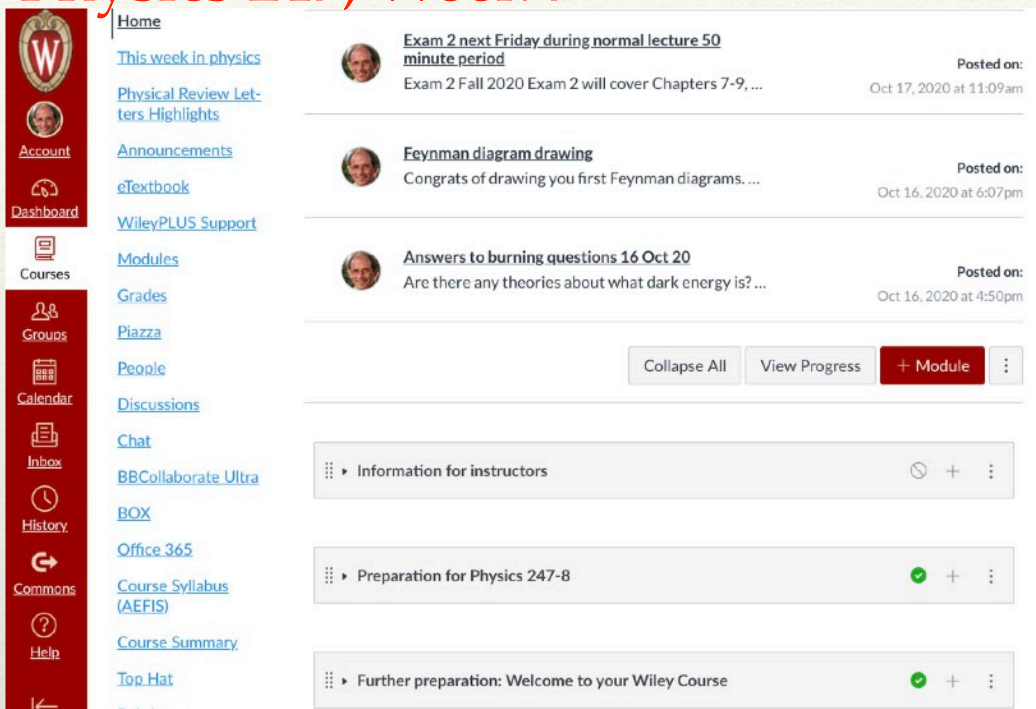
Week	Mon	MATLAB assignment Tutorial	Lab M 1:20 601, T 1:20 602, W 2:25 603, R 1:20 604 CH4136 Lab	Wed	Disc: W 1:20, 2:25 R 1:20, 2:25 Chamberlin 2104	Fri
1	no class	MATLAB onramp, MATLABGetting Started with MATLAB, MATLAB Introduction to MATLAB, MATLAB Image processing	No lab UW Instruction begins	Measurement, Ch. 1	Ch. 1,2	Motion Along a Straight Line Ch. 2
2	Vectors Ch. 3	MATLAB Concepts in probability,	Computer vision	Motion in 2 and 3 dimensions Ch. 4	Ch. 3,4	Motion in 2 and 3 dimensions Ch. 4
3	Force and Motion I Ch. 5	MATLAB quiver plot, MATLAB More concepts in probability	Acceleration and free fall	Force and Motion II Ch. 6	Ch. 5,6	Force and Motion II Ch. 6
4	Kinetic Energy and Work Ch 7	No MATLAB	No lab	Kinetic Energy and Work Ch 7	Ch. 6,7	Exam 1, CH1-6
5	Potential Energy and Energy Conservation Ch. 8	MATLAB Curve fitting	Computation and modeling	Potential Energy and Energy Conservation Ch.	Ch. 8,9	Center of Mass and Linear momentum Ch. 9
6	Center of Mass and Linear Momentum Ch. 9	MATLAB StackedBallCollisions	Inertial sensors	Relativity Ch. 37	Ch. 37	Relativity Ch. 37
7	Quarks, Leptons, and the Big Bang Ch. 44	MATLAB dubugging	Particle physics	Quarks, Leptons, and the Big Bang Ch. 44	Ch. 44	Rotation Ch. 10
8	Rotation Ch. 10	No MATLAB	No lab, formal lab report 1 due	Rolling, torque, and angular momentum Ch. 11	Ch. 9,10	Exam 2, CH6-9,37,44
9	Rolling, torque, and angular momentum Ch. 11	MATLAB Black Holes	Cosmic muons and radioactivity	Equilibrium Ch. 12	Ch. 11,12	Gravitation Ch. 13
10	Gravitation Ch. 13	MATLAB Ephemerides	Asteroids and bombs	Fluids Ch. 14	Ch. 13,14	Fluids Ch. 14
11	Oscillations Ch. 15	MATLAB Fit parametric curves	Simple harmonic oscillator and resonance	Oscillations Ch. 15	Ch. 15	Waves I Ch. 16
12	Waves I Ch. 16	No MATLAB	Standing waves on a string	Waves II Ch. 17	Ch. 16,17	Exam 3, CH10-15
13	Waves II Ch. 17	No MATLAB	No lab, formal lab report 2	Temperature, Heat and the first law of	Thanksgiving	Thanksgiving
14	Temperature, Heat and the first law of Thermodynamics	MATLAB LIGO Analysis	Acoustics and Doppler shift	The Kinetic Theory of Gases Ch. 19	Ch. 18,19	The Kinetic Theory of Gases Ch. 19
15	Entropy and the 2nd Law of Thermodynamics	MATLAB Exoplanets	No lab	Entropy and the 2nd Law of Thermodynamics	Ch. 20	Exam study period
						Exam 4, Ch 16-20 12/12/20, Saturday, 10:05AM - 12:05PM

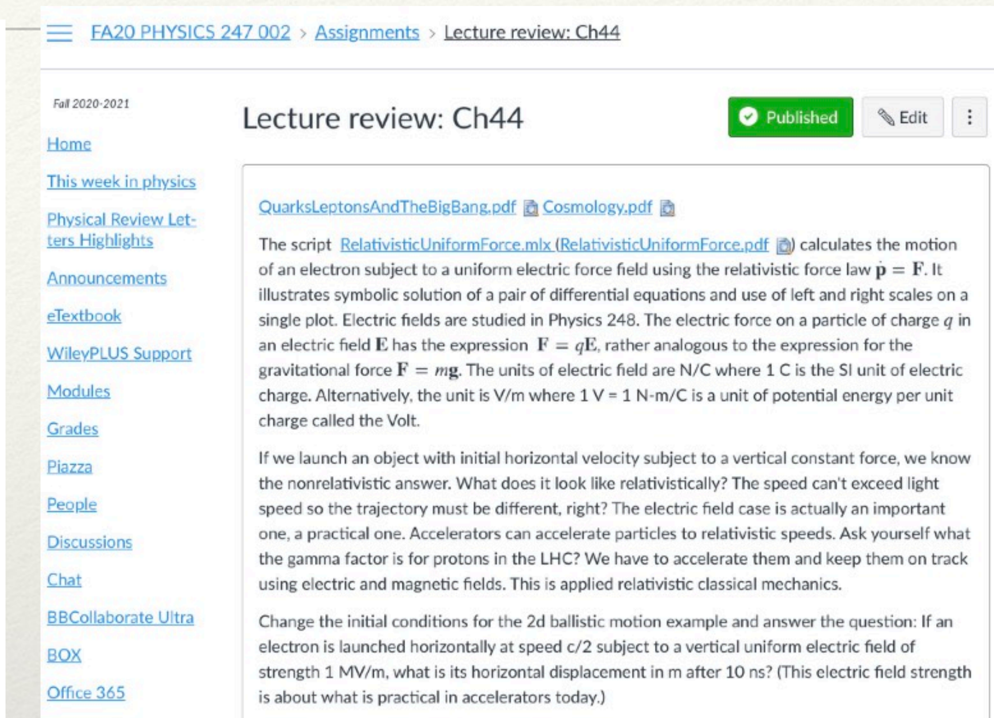
2nd semester syllabus

Week	Mon	MATLAB lab	Lab M or T 1:20-4:15 W 2:25-5:20,CH3136 Prelab	Wed	Disc: R 1:20 2:25, 3:30	Fri
1	no class	Tutorial	Lab	Coulomb's Law, Ch. 21	Group and challenge	Coulomb's Law, Ch. 21
2	Electric Fields, Ch. 22	MATLAB Vectors and rotations	Electric Fields	Electric Fields, Ch. 22	Group and challenge	Gauss' Law, Ch. 23
3	Gauss' Law, Ch. 23	Matlab Electric fields and potentials	Magnetic field map	Electric Potential, Ch. 24	Group and challenge	Electric Potential, Ch. 24
4	Capacitance, Ch. 25	MATLAB Electrostatic induction (and the	Oscilloscopes and RC circuits	Capacitance, Ch. 25	Group and challenge	Current and Resistance, Ch. 26
5	Current and Resistance, Ch. 26	MATLAB Method of images	Electron charge to mass ratio	Circuits, Ch. 27	Group and challenge	Circuits, Ch. 27
6	Magnetic Fields, Ch. 28	MATLAB Vector Calculus	No lab	Magnetic Fields, Ch.28	Group and challenge	Exam 1, CH 21-26
7	Magnetic Fields Due to Currents,	MATLAB Magnetic field of a coil	Magnetic induction	Magnetic Fields Due to Currents, Ch.	Group and challenge	Induction and Inductance, Ch. 30
8	Induction and Inductance, Ch. 30	MATLAB World's simplest electric train	LRC Circuit and Resonance	Electromagnetic Oscillations and	Group and challenge	Electromagnetic Oscillations and
9	Spring Break			Spring Break	Spring Break	Spring Break
10	Maxwell's Equations;Magneti	MATLAB ABCD Matrices	Mirrors and lenses	Maxwell's Equations;Magnetis	Group and challenge	Electromagnetic Waves, Ch. 33
11	Electromagnetic Waves, Ch. 33	MATLAB Diffraction	Optical Instruments	Images, Ch. 34	Group and challenge	Images, Ch. 34
12	Interference, Ch. 35	MATLAB Monte Carlo methods	No Lab	Interference, Ch. 35	Group and challenge	Exam 2, CH27-32
13	Diffraction, Ch. 36	MATLAB Bound states	Diffraction and Interference	Diffraction, Ch. 36	Group and challenge	Photons and Matter waves, Ch. 38
14	Photons and Matter waves, Ch.	MATLAB Quantum mechanics	Balmer Series	More About Matter waves, Ch. 39	Group and challenge	More About Matter waves, Ch. 39
15	All About Atoms, Ch. 40	MATLAB Electric field of an accelerated charge - 3d	Speed of light	All About Atoms, Ch. 40	Group and challenge	All About Atoms, Ch. 40

Tutorial: Relativistic motion

Physics 247, Week 7





QuarksLeptonsAndTheBigBang.pdf Cosmology.pdf

The script RelativisticUniformForce.mlx (RelativisticUniformForce.pdf) calculates the motion of an electron subject to a uniform electric force field using the relativistic force law $\dot{\mathbf{p}} = \mathbf{F}$. It illustrates symbolic solution of a pair of differential equations and use of left and right scales on a single plot. Electric fields are studied in Physics 248. The electric force on a particle of charge q in an electric field $\mathbf{E}$ has the expression $\mathbf{F} = q\mathbf{E}$, rather analogous to the expression for the gravitational force $\mathbf{F} = m\mathbf{g}$. The units of electric field are N/C where 1 C is the SI unit of electric charge. Alternatively, the unit is V/m where 1 V = 1 N-m/C is a unit of potential energy per unit charge called the Volt.

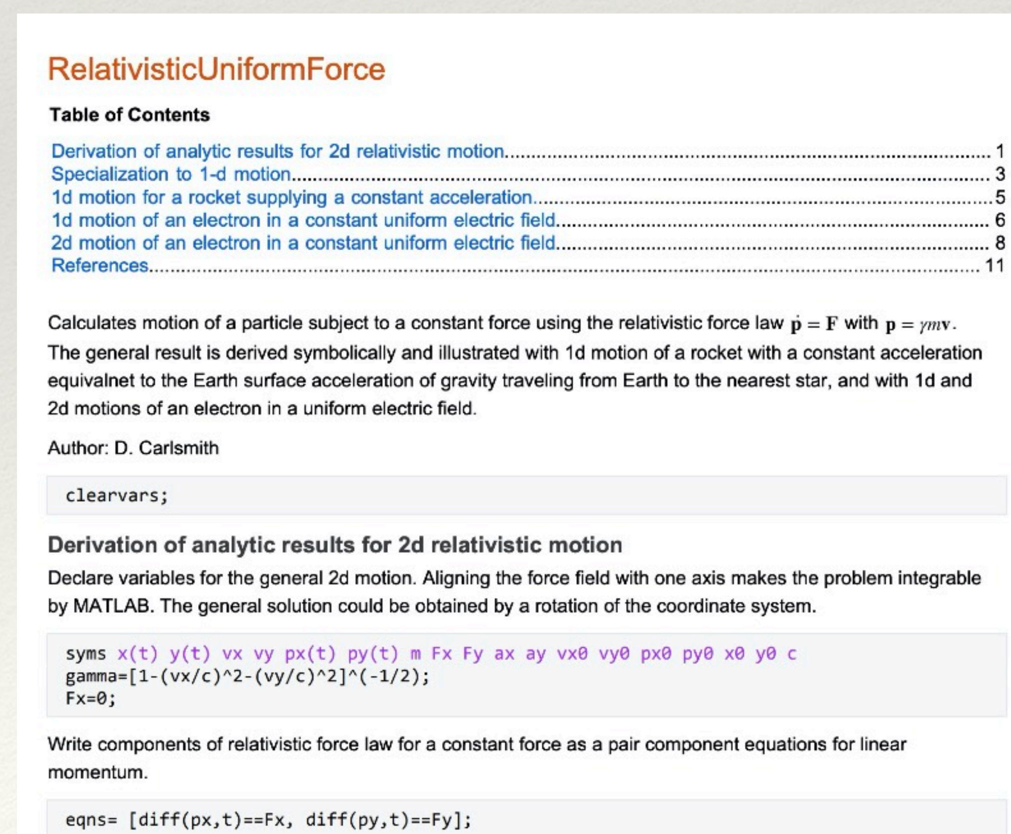
If we launch an object with initial horizontal velocity subject to a vertical constant force, we know the nonrelativistic answer. What does it look like relativistically? The speed can't exceed light speed so the trajectory must be different, right? The electric field case is actually an important one, a practical one. Accelerators can accelerate particles to relativistic speeds. Ask yourself what the gamma factor is for protons in the LHC? We have to accelerate them and keep them on track using electric and magnetic fields. This is applied relativistic classical mechanics.

Change the initial conditions for the 2d ballistic motion example and answer the question: If an electron is launched horizontally at speed c/2 subject to a vertical uniform electric field of strength 1 MV/m, what is its horizontal displacement in m after 10 ns? (This electric field strength is about what practical in accelerators today.)

Interactive Live Script

Module

Post lecture assignment



RelativisticUniformForce

Table of Contents

Calculates motion of a particle subject to a constant force using the relativistic force law $\dot{\mathbf{p}} = \mathbf{F}$ with $\mathbf{p} = \gamma m\mathbf{v}$. The general result is derived symbolically and illustrated with 1d motion of a rocket with a constant acceleration equivalnet to the Earth surface acceleration of gravity traveling from Earth to the nearest star, and with 1d and 2d motions of an electron in a uniform electric field.

Author: D. Carlsmith

```
clearvars;
```

Derivation of analytic results for 2d relativistic motion

Declare variables for the general 2d motion. Aligning the force field with one axis makes the problem integrable by MATLAB. The general solution could be obtained by a rotation of the coordinate system.

```
syms x(t) y(t) vx vy px(t) py(t) m Fx Fy ax ay vx0 vy0 px0 py0 x0 y0 c
gamma=[1-(vx/c)^2-(vy/c)^2]^(-1/2);
Fx=0;
```

Write components of relativistic force law for a constant force as a pair component equations for linear momentum.

```
eqns= [diff(px,t)==Fx, diff(py,t)==Fy];
```

❖ This example: Apply prior lab in symbolic and numerical modeling of systems of ode's (non relativistic realistic ballistic motion with drag in 2d) to relativistic motion.

Relativistic motion

- Relativistic Newton's 2nd law
$$\frac{d\mathbf{p}}{dt} = \frac{d}{dt}\frac{m\mathbf{v}}{\sqrt{1-\mathbf{v}^2/c^2}} = \mathbf{F}$$
- Simple case of hyperbolic motion $\mathbf{F} = \text{constant}$.
- Simple simple case: 1d motion, $v \to c$ asymptotically.
    - Application how long (proper time) would it take a human to reach the nearest start with a survivable (few g) rocket acceleration.

Practical application

- Static and dynamical charged particle accelerators have electric field gradients of order 1 MV/m over length scales of order 1 m.
- The electron mass is 0.511 MeV so electrons can be accelerated to relativistic speed in ~ 1 m.
- What does a 2d "ballistic trajectory" look like for such a particle?

MATLAB symbolic ode solution

- 6 lines of code to define symbols, equations, initial conditions, and solve for velocity $\mathbf{v}(t)$ and then solve for coordinate functions $\mathbf{x}(t)$.

MATLAB Live Script

RelativisticUniformForce

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Write components of relativistic force law for a constant force as a pair component equations for linear momentum.

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```

Equations

Define initial conditions.

```
cond=[px(0)==px0, py(0)==py0];
```

Conditions

Solve the pair of equations subject to the initial conditons using dsolve and catch a structure S containing the solutions.

```
S=dsolve(eqns,cond);
```

Solve

We use structure.thing-we-want notation to access the results in the structure S.

Try this: Look at the answer for py.

Solution

Extract the solutions for coordinates as functions of time.

```
y=Sx.y
```

```
y =
```

Thanks MATLAB!

$$y_0 + \frac{c\,\sqrt{\mathrm{Fy}^2\,t^2 + 2\,\mathrm{Fy}\,\mathrm{py}_0\,t + c^2\,m^2 + {\mathrm{px}_0}^2 + {\mathrm{py}_0}^2}}{\mathrm{Fy}} - \frac{c\,\sqrt{c^2\,m^2 + {\mathrm{px}_0}^2 + {\mathrm{py}_0}^2}}{\mathrm{Fy}}$$

```
x=Sx.x
```

```
x =
```

$$x_0 + \frac{c\,\mathrm{px}_0\,\log\left(\sqrt{\mathrm{Fy}^2\,t^2 + 2\,\mathrm{Fy}\,\mathrm{py}_0\,t + c^2\,m^2 + {\mathrm{px}_0}^2 + {\mathrm{py}_0}^2} + \frac{t\,\mathrm{Fy}^2 + \mathrm{py}_0\,\mathrm{Fy}}{\mathrm{Fy}}\right)}{\mathrm{Fy}} - \frac{c\,\mathrm{px}_0\,\log\left(\mathrm{py}_0 + \sqrt{c^2\,m^2 + \mathrm{p}}\right.}{\mathrm{Fy}}$$

The expressions x, y, Vx, Vy give the coordinate and velocity vectors in terms of initial values for coordinates and momentum. We could substitute expressions for momentum components in terms of velocity components to express position and velocity components interms of initial position and initial velocity components using subs.

Symbolic sanity checks

- Work energy theorem
- Approaching light speed as $t \to \infty$

Compare change in kinetic energy to the work done. These quantities should be equal.

```
simplify(KE-subs(KE,t,0)-Fy*(y1d-y0),'IgnoreAnalyticConstraints',true)
```

ans = 0

Compute position and velocity for force equivalent to accleration of gravity starting from rest.

```
syms g
vars=[y0,py0,Fy];
vals=[0 ,0,  m*g];
yg=simplify(subs(y1d,vars,vals),'IgnoreAnalyticConstraints',true)
```

yg =

$$-\frac{c\left(c-\sqrt{c^2+g^2\,t^2}\right)}{g}$$

Note, as $t \to \infty$, $yg \to ct$. We can demonstrate this with limit.

```
assume(g>0);assume(c>0)
limit(yg/t,t,Inf)
```

ans = c

```
vyg=simplify(subs(vy1d,vars,vals),'Steps',2,'IgnoreAnalyticConstraints',true)
```

vyg =

$$\frac{c\,g\,t}{\sqrt{c^2+g^2\,t^2}}$$

Try this: Demonstrate that as $t \to \infty$, $vyg \to c$ using `limit`.

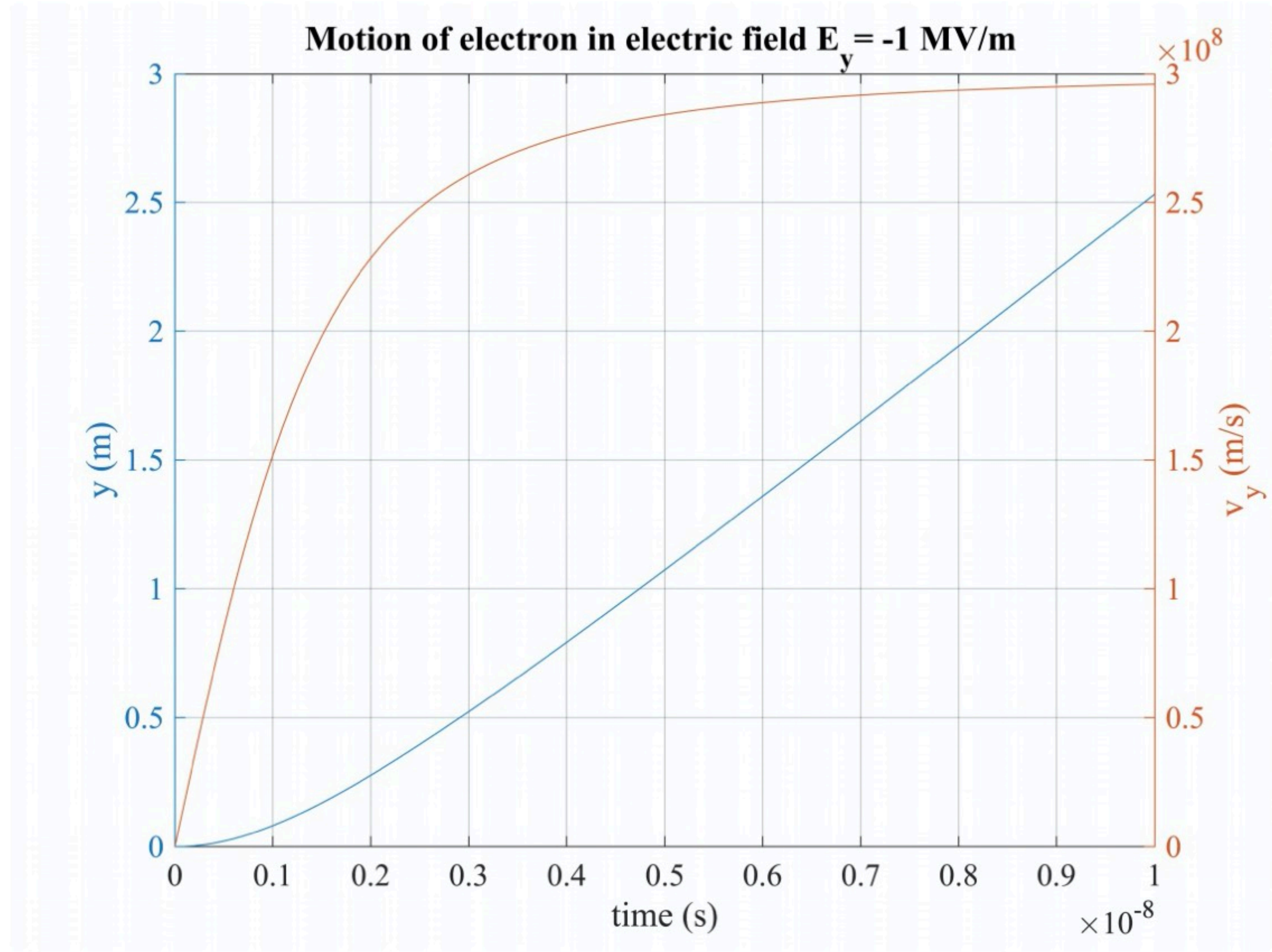
Compute the time to reach a distance L.

```
syms L real
assume(L>0);assume(g>0);assume(yg>0);assume(c>0);assume(t>0);assume(m>0)
SL=solve(yg==L,t,'IgnoreAnalyticConstraints',true);
```

“Try this” prompt for student to exercise MATLAB functions.

Application to 1d electron motion

- Notice $v_y \to c$ and displacement becomes a linear function of time.



Notice in 1 m of displacement the electron gains a kinetic energy of 1 MeV or about twice its rest energy fromt he work done by the electrical force. The velocity at that point is given by the following calculation.

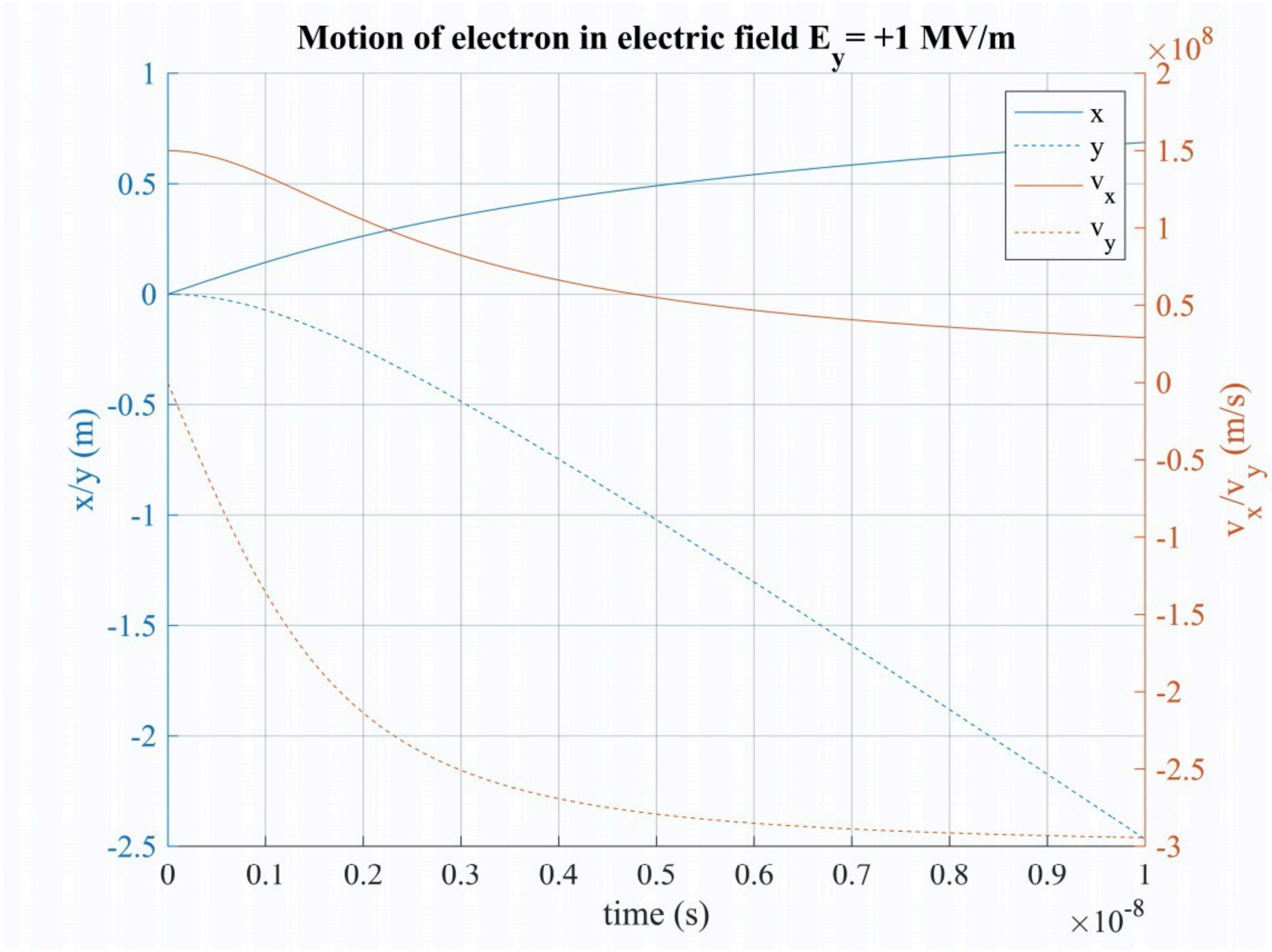
```
yDisplacement=1;
E=mValue*cValue^2+e*Efield*yDisplacement;
```

$E = mc^2/\sqrt{(}1 - v^2/c^2) \to 1 - v^2/c^2 = (mc^2/E)^2 \to v/c = \sqrt{1 - (mc^2/E)^2}$

```
v=cValue*sqrt(1-(mValue*cValue^2/E)^2)
```

Application to 2d motion

- The velocity component perpendicular to the force field changes!



Notice that the x-component of velocity, subject to no force, nevertheless changes - the x-component of momentum is constant but the speed is changing due to acceleration in the y direction - and approaches zero as $v_y \to -c$.

Proper time to get to a star

- Given time dilation, it is possible in principle to travel to a star in an arbitrarily short proper time. (Though your twin will age much faster.)
- But if you accelerate from rest without blood squirting out your eyeballs, how much time would it actually take?
- Compute for a~g arrival time and then proper time for L~ 4 LY.

Compute the proper time $\tau = \int_0^t dt/\gamma(t)$ using int. This is the time evolved by a traveler during the trip according to the traveler.

```
gamma_g=(1-(vyg/c)^2)^(-1/2)
```

gamma_g =

$$\frac{1}{\sqrt{1-\frac{g^2\,t^2}{c^2+g^2\,t^2}}}$$

```
tau_g=int(simplify(1/gamma_g),t,'IgnoreAnalyticConstraints',true )
```

tau_g =

$$\frac{c\,\mathrm{asinh}\left(\frac{g\,t}{c}\right)}{g}$$

Thanks for listening